

OPTIMAL LIFETIME UTILITY WHEN TIME PREFERENCE DEPENDS ON SURVIVAL PROBABILITIES

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Abstract

We provide a structural theory of time preference and derive a functional form of intertemporal preferences by postulating that individuals make their life-cycle choices as if to maximize expected lifetime. This approach yields a non-time-separable expected utility representation. The rate of time preference is found to depend on the inverse of expected remaining lifetime and on the effect of age on the productivity of consumption in affecting health. Preferences display nearby complementarity and are such that the inverse of the coefficient of intertemporal substitution exceeds the coefficient of relative risk aversion.

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1. Introduction

Economic theory has traditionally regarded preferences as given. As a result, there is little guidance for economists on how to formulate intertemporal preferences. The standard approach, derived from Samuelson (1937), is to assume time-separable preferences with a constant rate of time preference. The purpose of this paper is to provide a structural theory of intertemporal utility in which the dynamic specification and the rate of time preference are endogenous.

We assume that individuals maximize their expected life span. Incorporating physical and economic constraints allows us to replace the metaphysical concept of a utility function with the observable concept of a “health function.” Intertemporal preference is accordingly viewed as the manifestation of whatever design of allocating consumption over time as an input to the health function maximizes expected survival time.

The assumption of maximization of expected life span is consistent with two different evolutionary perspectives. First, the biological view. In line with a growing body of literature, we may view preferences as the end product of natural selection: subject to physiological constraints, preferences that survive maximize some measure of fitness. Fitness is typically operationalized as number of offspring raised. For instance, Maynard Smith (1982) considers maximization of expected offspring as the individual’s objective. We adopt the similar but simpler objective of expected life span maximization to focus more directly on time preference issues.¹ A second evolutionary perspective motivating expected life span maximization is related to cultural

¹As evolution is chasing a moving target due to the changing environment, the principle of “gene inertia” arises. For the human species this principle implies that impulses designed to maximize fitness in the once-stable hunter-gatherer environment, are the impulses that apply today. With a life span typically not exceeding forty years, increases in lifetime in hunter-gatherer times may be identified on a one-to-one basis with increases in reproductive years. Additionally, if a general monotonic relationship between lifetime and offspring is assumed, a change-of-variable establishes (results available from the authors) that the expected offspring maximization problem can be reduced to one of maximizing expected lifetime in units of “reproductive time” as long as the health function (to be discussed) is appropriately modified.

learning. Children learn from their parents' generation how to live; the life styles that lead to increased life expectancy are more likely to be imitated (either by direct parental guidance or the children's choice).

The impact of evolutionary selection or survival on intertemporal choice has not received much attention in the literature. Yaari (1965) allows the rate of time preference to vary based on the probability of death; this probability, however, is exogenous.² Rogers (1994) applies an idea in Hansson and Stuart (1990), where the marginal rate of substitution in preferences is set equal to the marginal rate of substitution in "fitness," to an intertemporal context. His paper has a role for bequests and specific implications for how an individual's time preference varies with age. It, however, takes a time-additive utility specification as given. Recently, Becker and Mulligan (1997) have provided a basic theory of time preference, which is not survival based. They assume that individuals may invest to increase their appreciation of the future, thus endogenously affecting their rates of time preference. As in Rogers, a drawback of the Becker and Mulligan formulation is that it assumes the additively-separable intertemporal utility form.

Our theoretical approach derives an intertemporal utility specification that sheds a preliminary theoretical light on debates concerning expected utility; time consistency; separability of consumption decisions; the difference between risk aversion and intertemporal substitution; the effect of health on life-cycle choices; and the factors governing time preference. Section 2 derives the intertemporal utility specification based on survival-time maximization. Section 3 considers the properties of the derived utility specification. Section 4 concludes the paper.

² The uncertain lifetime formulation has been applied by Barro and Friedman (1977), Levhari and Mirman (1977), Davies (1979), and Blanchard (1985).

2. Derivation of the Intertemporal Utility Function

The key assumption in the paper is to impute the maximization of expected lifetime as an individual's sole lifetime goal.³

***Assumption 1.** An individual's lifetime objective is to maximize $E(T | \cdot)$, where T is the time of the individual's death.*

Formally, we define a memory-less continuous-time two-state Markov Chain $X(t) : \mathbb{R}^+ \rightarrow S$, with $S = \{0, 1\}$. The expectation is taken contingent on the current transitory state $X(0) = 1$; Death is defined as the absorbing state $X(t) = 0$. Thus we define the expected lifetime $E(T | \cdot)$ as the mean time until absorption given that $T = \inf \{t : X(t) = 0 | X(0) = 1\}$.

Denote $C(t) \equiv \{c(s) : 0 \leq s \leq t\}$ as the consumption history until time t , where $c(s) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ indicates consumption at time s , and $\lim_{t \rightarrow \infty} C(t) \equiv C$ as the infinite horizon consumption path. Then define: $G[t | C(t)] \equiv Pr[T \leq t | C(t)] = Pr[X(t) = 0 | X(0) = 1, C(t)]$ as the probability distribution of being dead by age t given consumption path $C(t)$, and $g[t | C(t)]$ as the associated density.⁴ Straightforward derivation yields that the *health-hazard*

³Assumption 1 here should be contrasted with the assumption made by Karni and Schmeidler (1986) for the purpose of examining risk attitudes: maximization of the end-of-period survival probability. Our approach is different in assuming maximization of expected survival time which we believe is more appropriate for the purpose of studying intertemporal preferences. Our approach is similar to Karni and Schmeidler's only in modeling competition of the individual against nature while ignoring explicit competition against other individuals.

⁴Ray and Streufert (1993) also assume that survival probabilities depend on the path of consumption but for a time-additive utility function.

$$\text{rate} \quad \lambda[C(t), t] \equiv \lim_{h \downarrow 0} \frac{Pr[X(t) = 0 | X(t-h) = 1, C(t)]}{h}$$

--the instantaneous probability of dying, *having already lived until time t*--equals

$$\lambda[C(t), t] = \frac{g[t | C(t)]}{1 - G[t | C(t)]} = \frac{-d \ln \{1 - G[t | C(t)]\}}{dt} . \quad (1)$$

Integrating and taking the antilog on both sides produces

$$1 - G[t | C(t)] = e^{-\int_0^t \lambda[C(s), s] ds} \quad (2)$$

Next relate expected lifetime to the hazard rate. Using integration by parts:

$$E(T|C) = \int_0^\infty t g[t | C(t)] dt = \int_0^\infty \{1 - G[t | C(t)]\} dt. \quad (3)$$

Combination of equations (2) and (3) produces the intertemporal utility function based on the impersonal evolutionary process that leads to maximization of expected lifetime.

Result 1. *Given Assumption 1, an individual's lifetime utility is given as the maximum of*

$$U(C) \equiv E(T|C) = \int_0^\infty e^{-\int_0^t \lambda[C(s), s] ds} dt . \quad (4)$$

Here T represents time of death and $\lambda[C(t), t]$ indicates the health-hazard rate at age t , given the consumption stream up to time t .

3. Properties of the Derived Utility Specification.

If the health-hazard rate at t is a function of instantaneous consumption at t only, then this derived utility function belongs to the family axiomatized by Epstein (1983) and thus is a von Neumann-Morgenstern utility function.⁵

To see this more generally in our case, when the health-hazard rate is a function of the stream of consumption until time t as well as of time t itself, we introduce uncertainty other than the hazard of death to shift focus temporarily from considering intertemporal preferences to considering risk preferences. Consider a continuum of possible states belonging to the state space $\Omega \subset \mathbb{R}$. In the above-discussed case where implicitly the state is assumed known, say equal to ω , the agent will determine an infinite horizon consumption plan $C(\omega)$ yielding certain utility $U[C(\omega)] \equiv E[T \mid C(\omega)]$. (Notice that U is deterministic for given ω , even when T is not). When the state is unknown, the (evolutionary) objective is still to maximize expected lifetime, $E(T \mid C)$. But, mathematically, $E(T \mid C) = E\{E[T \mid C(\omega)]\} = E\{U[C(\omega)]\}$, where the expectation in the final expression is taken over all $\omega \in \Omega$. We thus have a von Neumann-Morgenstern utility function.

The assumptions needed for the expected utility property (such as the independence axiom) are embedded in the objective we chose to operationalize the concept of survival (assumption 1). Our contribution is that the assumed linearity in the probabilities in the objective function is not arbitrary in that it is the expected lifetime, resulting from a particular behavioral pattern, that matters. It is outside the scope of the current paper -- focusing on intertemporal

⁵Bergman (1985), Epstein and Hynes (1983), Obstfeld (1990) and Uzawa (1963) also discuss members of the class axiomatized by Epstein (1983). These authors however do not examine the particular form that we derive here.

preference -- to mathematically justify the assumption of expected lifetime maximization from even more basic principles.⁶

Robson (1996), considering the risk attitudes deriving from an evolutionary process, reaches the same conclusion, supporting expected utility, when risk is idiosyncratic. However, he goes back further in *deriving* the validity of the expected offspring criterion used by Maynard Smith (1982) and others (including us).⁷

Returning to issues of dynamic preference specification, and accordingly dropping for simplicity all uncertainty other than the hazard of death, we now show that our utility function in equation (4) also implies time consistency. Factoring the right-hand side of equation (4) yields:

$$\int_0^{\infty} e^{-\int_0^{\tau} \lambda[C(s), s] ds} d\tau = \int_0^t e^{-\int_0^{\tau} \lambda[C(s), s] ds} d\tau + e^{-\int_0^t \lambda[C(s), s] ds} \left(\int_t^{\infty} e^{-\int_t^{\tau} \lambda[C(s), s] ds} d\tau \right)$$

Or, using equation (4) both for the first and for the last term,

$$E(T|C) = a[C(t), t] + b[C(t), t] E(T - t | C, t) ,$$

where the conditioning information t is shorthand for $X(t) = 1$ so that

⁶An evolutionary motivation, however, is that the independence axiom (considered the most controversial of the axioms guaranteeing the expected utility property) should hold since taking account of the realizations of unrealized alternatives is unproductive and has no survival value: the survival value attached to a particular consumption path should not depend on what would happen if another consumption path were realized, as we implicitly assume via assumption 1.

⁷Robson additionally finds that selection in the context of aggregate shocks may invalidate the expected offspring criterion: the effect of a shock on an individual is different when all members of a particular risk-attitude type are affected by the shock than when instead only the individual is affected (the aggregate risk lowers the expected growth rate more than the idiosyncratic risk). While Robson interprets this result as generating non-expected utility behavior (the axiom of reduction of compound lotteries breaks down, not the independence axiom) one could alternatively consider expected-utility maximizing individuals as endowed with altruistic feelings towards their group, causing them to be more risk averse in the face of aggregate risk.

$E(T|C) \equiv E(T-0 | C, 0)$. Thus, the specific form here allows us at each point in time t to maximize $E(T-t | C, t)$. Clearly, the decisions based on the continuation at time t of the policy based on preferences at time 0 are equivalent to the decisions made at time t based on preferences at time t -- anticipated preference reversals do not occur and preferences are time consistent:

$$\underset{\{C(s)\}_{t}^{\infty}}{\operatorname{argmax}} E(T|C) = \underset{\{C(s)\}_{t}^{\infty}}{\operatorname{argmax}} E(T-t | C, t)$$

Deaton (1992, p.15) states that time inconsistent preferences are irrational. This is not obvious given a typical view of rationality as “behavior consistent with the objectives”--complex objectives may, in principle, allow for any type of behavior. To the extent that survival mechanisms can only support objectives like maximization of expected lifetime, our approach provides a rationale for marking time inconsistent behavior as irrational.

A further property -- that preferences are not time separable -- follows directly from equation (4). It is straightforward to show that the marginal utility of consumption at time t depends on future levels of consumption (as will be apparent from equation 5, for instance). The absence of time-separability is consistent with the opinions of many (for instance Lucas, 1978, p. 1444) that there is no rationale other than convenience for assuming a time-additive utility specification. Uzawa (1968) previously considered, without derivation, a non-time-separable form related to ours but with the rate of time preference given *ad hoc*, as a function of consumption in different periods. In Ryder and Heal (1973) the utility function is assumed to depend on a weighted average of past consumption levels, with weights declining exponentially

into the past.⁸

The above consequences are summarized as follows:

Result 2. *The preference specification $U(C)$ in equation (4) consistent with maximizing expected lifetime according to Assumption 1: (a) has the expected utility property; (b) is time consistent; and (c) is not time separable.*

Further properties of the derived utility functional in equation (4) will be obtained under a simpler specification of the health-hazard rate.⁹

Assumption 2. *For all $t \in [0, \infty)$ the hazard rate depends only on age t and current consumption $c(t)$: $\lambda[C(t), t] = \lambda[c(t), t]$. It is twice continuously differentiable and a positively valued, negatively sloped, and strictly convex function of $c(t)$.*

Given equation (4) and the assumed hazard rate specification, a change in consumption at time t implies the following Volterra derivative denoted by “ ’ ” (for a similar use of the Volterra derivative see, for instance, Ryder and Heal, 1973, and Epstein and Hynes, 1983):

$$U'(t) = -\lambda_c[c(t), t] (1 - G[t|C(t)]) E(T-t|C, t), \quad (5)$$

$$\text{where } (1 - G[t|C(t)]) E(T-t|t) = \int_t^\infty e^{-\int_0^\tau \lambda[c(s), s] ds} d\tau.$$

⁸More recent applications of the non-time-separable utility specification include the work of Bergman (1985), Obstfeld (1990), and Shi and Epstein (1993).

⁹In a more general specification, the health hazard could also depend on the lagged health hazard to add realism. However, our specific and methodological points are most easily made using the simpler specification of assumption 2.

As in the following, subscripts represent partial derivatives. Thus $-\lambda_c$ indicates the marginal impact of additional consumption in reducing the health hazard, which is multiplied by the probability of being alive at time t and expected remaining lifetime, the latter representing the loss at sudden death.

Some basic properties of the utility functional follow readily from equation (6). Consider a discrete-time version of equation (4). Since, as follows from equation (5), utility is monotonically increasing in each of its arguments, it follows directly that the discrete version of the utility specification in equation (4) is monotonic and, thus, quasi concave. The continuity of $\lambda(\cdot)$ from Assumption 2 assures that, in real space, the continuous-time utility functional is quasi concave as well. As a result, the indifference curves for consumption at two separate instances in time are convex:

Result 3. *Given Assumption 2, the utility functional in equation (4) is monotonic and quasi concave, and has convex indifference curves.*

Next define the discount rate following Epstein and Hynes (1983) as,

$$\rho(t) = \frac{U'(t-h)}{U'(t)} - 1 = \frac{-\partial \ln U'(t)}{\partial t} \quad (6)$$

with $c(t) = c(t-h)$ and for $h \downarrow 0$. Equation (6) captures the basic notion of time preference in continuous time: all else (i.e., consumption) equal, by which fraction is the marginal utility from consumption at some point in time, $t-h$, higher than the marginal utility from consumption at a slightly later point in time, t . This time preference concept is a marginal concept -- applying to

time preference at a point in time. As time preference may change over time, the average concept -- considering the rate of time preference over a longer period -- will in general be different.

Differentiating the log of the right-hand side of equation (5) with respect to time t , equation (6) yields after some cancellations:

$$\rho(t) = \left[\int_t^\infty e^{-\int_t^T \lambda[c(s), s] ds} dT \right]^{-1} - \frac{\lambda_{ct}[c(t), t]}{\lambda_c[c(t), t]}. \quad (7)$$

Equation (7) implies,

Result 4. *Given Assumptions 1 and 2, an individual's rate of time preference (discount rate) is equal to*

$$\rho(t) = \frac{1}{E(T - t | C, t)} - \frac{\lambda_{ct}[c(t), t]}{\lambda_c[c(t), t]}. \quad (8)$$

With t a particular time period during which the individual is alive.

The derivation follows from equation (7) and the definition in equation (4). Assuming that the productivity of consumption in affecting health is constant with age, individuals with a higher life expectancy have a longer horizon and thus should put more weight on future events. Taking the result a little more seriously than is intended, one may obtain the numerical value for the average rate of time preference from equation (8), when λ_{ct} is set equal to zero. Based on the instincts

surviving from hunter-gatherer times, the conditional life expectancy of the average individual living through early childhood may lie around 30 or 35 years left to live. The rate of time preference from equation (8) should then be around 3%, which appears to be in the ballpark compared to the actual numbers.¹⁰ Again assuming constant health effects of consumption with age (or controlling for age), some confirmation of result 4 is provided by Leigh (1986) who finds that -- controlling for income -- African Americans, who as a group have a lower life expectancy, also have a significantly higher rate of discount (a result confirmed by Cropper et al., 1994).

Some further results easily follow from result 4:

Result 5. *Given Assumptions 1 and 2, if the effect of consumption on health is independent of age, wealthier individuals cannot have a higher rate of time preference.*

Proof. An increase in wealth cannot decrease an individual's maximum utility level. Thus, from equation (1), expected lifetime rises (or remains unchanged) which lowers (or maintains) the rate of time preference from equation (8) when λ_{ct} equals 0.

Given result 5 it is easy to imagine a cycle of poverty. As an individual becomes poorer this individual also rationally becomes more myopic, leading to relatively higher consumption, exacerbating the degree of poverty. Result 5 is confirmed empirically by Lawrance (1991) and Viscusi and Moore (1989). Lawrance, using panel data, finds a rate of time preference of poorer households that is three to five percent higher than that of wealthier households. Viscusi and Moore find that households with lower earning potential (lower life-time wealth) have a higher

¹⁰Rogers (1994) obtains a slightly lower number based on population growth, average generation length, and the fraction, 0.5, of shared genes between parent and offspring.

rate of time preference than those with higher earning potential.

A further result provides the circumstance under which the rate of time preference equals the hazard rate as assumed for instance in Blanchard (1985).

Result 6. *If consumption is constant and health does not depend on age then the rate of time preference is constant and equal to the health-hazard rate.*

Proof. In equation (7) keep consumption constant and pull through the integral to obtain:

$$\rho(t) = \left[\int_t^\infty e^{-\lambda(\tau-t)} d\tau \right]^{-1} = \left[\int_0^\infty e^{-\lambda\tau} d\tau \right]^{-1} = \lambda. \quad (9)$$

The last equality holds since the term in square brackets represents the expected value of the exponential distribution with parameter λ .

The standard time-additive preference specification implies lack of intertemporal dependence: consumption choices at a particular time are independent of consumption realizations at other times. We examine next to what extent intertemporal dependence in consumption exists for our specification. Following the approach in Ryder and Heal (1973) define the marginal rate of substitution between consumption at time t_1 and t_2 (with $t_1 < t_2$) as: $M(t_1, t_2; C) = U'(t_1)/U'(t_2)$. Then we may check if changes in consumption at time s affect the marginal rate of substitution. Taking the Volterra derivative of $M(\cdot)$ with respect to a change in consumption at time s yields:

$$M'(t_1, t_2; C, s) = \frac{U''(t_1, s)U'(t_2) - U''(t_2, s)U'(t_1)}{[U'(t_2)]^2}.$$

Using equations (5) and (A2) produces:

$$M'(t_1, t_2; C, s) = \frac{-\lambda_c[c(t_1), t_1] \lambda_c[c(s), s]}{\lambda_c[c(t_2), t_2] [H(t_2)]^2} [H(t_2) H(\tau_1) - H(t_1) H(\tau_2)], \quad (10)$$

$$\text{with: } \tau_i \equiv \max\{t_i, s\}, \quad H(t) \equiv [1 - G(t|C)] E(T - t|C, t).$$

Note that the term outside brackets on the right-hand-side of equation (10) is positive. Figure 1 displays how $M'(\cdot)$ changes depending on time s relative to times t_1 and t_2 . For $s > t_2$ we have $M'(\cdot) < 0$ -- implying “nearby complementarity.” Recall from equation (5) that the marginal benefit of consumption at age t is proportional to the probability of survival until time t times the expected remaining lifetime from time t on. Nearby complementarity thus occurs since an increase in future consumption raises current expected remaining lifetime more if the time of the future consumption increase is closer -- the benefit of the future consumption increase is discounted less. For $s < t_1$, when consumption benefits at t_1 relative to t_2 are considered, the increased consumption level at time s is a bygone and so $M'(\cdot) = 0$. In general, from equation (10), as shown in Figure 1:

Result 7. Consider, without loss of generality, $t_2 > t_1$. For time $s > [\leq] t_1$, an increase in consumption at s lowers [leaves unaffected] the marginal rate of substitution between consumption at t_1 and t_2 .

Intuitively, one might expect that “distant complementarity” would result, at least in some situations, since more distant consumption is typically preferred over immediate consumption right after a “heavy meal.” The reason that our utility functional does not produce such a result lies in assumption 2, which rules out health benefits of lagged consumption. If we were to model the health benefits of consumption as based on a consumption stock (consisting of a summation of properly depreciated past consumption), then our approach would yield a motivation from first principles for the Ryder and Heal (1973) specification. Accordingly, if allowance for a consumption stock is made in the model, distant complementarity may occur if consumption levels at times t_1 and t_2 are high since, with a high current consumption stock, postponing future consumption to a time where it may be more valuable at the margin could be optimal.

A different limitation of the standard time-separable specification of utility is that the coefficient of risk aversion must equal the inverse of the elasticity of substitution. In the appendix we derive that

$$R[c(t), t] = \frac{c \{ \lambda_{cc}[c(t), t] - \lambda_c^2[c(t), t] \}}{-\lambda_c[c(t), t]} \quad (11)$$

where $R[c(t), t]$ represents the coefficient of relative risk aversion at a consumption level for time t . Defining $\sigma[c(t), t]$ as the coefficient of intertemporal substitution, the appendix obtains

$$\frac{1}{\sigma[c(t), t]} = \frac{c \lambda_{cc}[c(t), t]}{-\lambda_c[c(t), t]} \quad (12)$$

Straightforward comparison of equations (11) and (12) produces the following result:

Result 8. *The inverse of the coefficient of intertemporal substitution exceeds the coefficient of relative risk aversion.*

The intuition is that the coefficient of intertemporal substitution captures the incentive to smooth the hazard of death over time, which depends on the curvature of the hazard rate. The coefficient of relative risk aversion on the other hand relates to the curvature of overall utility which is less than the curvature of the hazard rate: a decrease in consumption at t lowers expected lifetime which dampens the overall effect on marginal utility due directly to a higher hazard of death at t (because the opportunity cost of death is equal to expected remaining lifetime).

An example may help to illustrate some of the advantages of the approach. Consider an isoelastic hazard rate, $\lambda(c, t) = [\alpha(t)/\gamma(t)]c^{1-\gamma(t)}$, where $\gamma(t) > 1$ is required for convexity. Then $1/\sigma(c, t) = \gamma(t)$ and $R(c, t) = \gamma(t) - \alpha(t)c^{1-\gamma(t)}$. Changes in the parameter path $\alpha(t)$ affect risk aversion without affecting intertemporal substitution. In this example risk aversion decreases as consumption falls since $\gamma(t) > 1$. For very low levels of consumption it even pays to seek risk and gamble. Seeking risk may be optimal in desperate situations, but it is easy to show that positive risk aversion can be guaranteed for all consumption levels if the health-hazard rate is equal to any monotonically increasing, concave transformation of the function $\gamma - \ln(c - \alpha)$ for $\alpha, \gamma > 0$.

4. Conclusion

We have provided a theoretical basis for dynamic utility specifications. The survival-oriented rationality of the individual objective function implies time consistency; and the derived utility function, although recursive, is not time-separable. It must be of the von Neumann-

Morgenstern variety, however. The theory provides insights into life-cycle choices by showing that the rate of time preference varies in an intuitive manner with changes in conditional lifetime, initial wealth, age, and the marginal productivity of consumption in affecting health.

Kacelnik (1998) states: “Neither animals nor humans are likely to be driven directly by the maximization of fitness, but we may understand the psychological mechanisms that do control their behaviour by asking about the fitness consequences of different courses of actions.” This statement characterizes our basic approach. Operationalizing “maximization of fitness” as we do in terms of maximization of expected lifetime, however, has some important shortcomings as an evolutionary motivation: it ignores the trade-off between survival and fertility as well as strategic interactions between individuals. Extending our approach to address these simplifications may yield further interesting results.

Explanation of standard anomalies may of course require other extensions of the approach, for instance by altering assumption 2 to allow the consumption history to affect current health. Take the observation that individuals prefer to delay pleasant events and like to accelerate unpleasant events as discussed by Loewenstein (1987). The survival-based explanation would be that an individual currently in good health would prefer to deal with unpleasant (i.e., potentially hazardous to life) events quickly, when bad outcomes can easily be absorbed; whereas pleasant events should be postponed so that they may benefit the individual at a potentially vulnerable time.

Appendix

A. Derivation of Equation (11)

We can take the Volterra derivative of equation (5) to obtain

$$U''(t) = -\lambda_{cc}(c, t) \int_t^\infty e^{-\int_0^\tau \lambda(c, s) ds} d\tau + [\lambda_c(c, t)]^2 \int_t^\infty e^{-\int_0^\tau \lambda(c, s) ds} d\tau. \quad (A1)$$

Again recalling (5), the standard Arrow-Pratt measure of relative risk aversion, using (A1), is

$$R(c, t) \equiv \frac{-U''(t) c}{U'(t)} = \frac{c[\lambda_c^2(c, t) - \lambda_{cc}(c, t)] \int_t^\infty e^{-\int_0^\tau \lambda(c, s) ds} d\tau}{-\lambda_c(c, t) \int_t^\infty e^{-\int_0^\tau \lambda(c, s) ds} d\tau}$$

Canceling the integral expressions produces equation (11) in the text.

B. Derivation of Equation (12).

The elasticity of the marginal rate of substitution between consumption at time t_2 and t_1 (with $t_2 > t_1$) can be expressed [see Silberberg (1990), p. 288] as:

$$\sigma \equiv \frac{-U'(t_1)U'(t_2)[U'(t_1)c(t_1) + U'(t_2)c(t_2)]}{c(t_1)c(t_2) [U'(t_2)^2 U''(t_1) - 2U'(t_1)U'(t_2)U''(t_1, t_2) + U'(t_1)^2 U''(t_2)]}$$

Taking Volterra derivatives based on equation (5) for $t_2 > t_1$:

$$U''(t_1, t_2) = \lambda_c(c, t_1) \lambda_c(c, t_2) \int_{t_2}^{\infty} e^{-\int_0^{\tau} \lambda(c, s) ds} d\tau \quad (\text{A2})$$

Let $t_2 \downarrow t_1$ so that, for continuous $c(t)$, $c(t_1) \rightarrow c(t_2)$ to ensure that $U'(t_1) \rightarrow U'(t_2)$. Then (A2)

becomes

$$U''(t, t) = \lambda_c^2(c, t) \int_t^{\infty} e^{-\int_0^{\tau} \lambda(c, s) ds} d\tau.$$

The elasticity of substitution then equals:

$$\sigma[c(t), t] = \frac{2U'(t)^3 c(t)}{2[U'(t)c(t)]^2 [U''(t) - \lambda_c^2(c, t) \int_t^{\infty} e^{-\int_0^{\tau} \lambda(c, s) ds} d\tau]}$$

Using equation (5) and (A1) yields the inverse of equation (12):

$$\sigma[c(t), t] = \frac{-\lambda_c[c(t), t]}{c(t) \lambda_{cc}[c(t), t]}$$

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Figure 1: Nearby Complementarity

